Unit Commitment with uncertainties - State of the art

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RESUME – L'insertion des énergies renouvelables (EnR) variables pose de nouveaux défis aux gestionnaires de réseaux (GR) pour concilier l'optimisation économique, la sécurité et la qualité de fourniture. Aujourd'hui, l'éolien et le photovoltaïque (PV) sont les EnR avec la plus forte croissance, mais elles s'accompagnent d'une variabilité peu contrôlable et d'une imprévisibilité partielle de leur production, ce qui impacte, entre autres, la gestion du parc de production. Ce travail présente différentes méthodes pour la prise en compte de ces incertitudes dans le placement de la production à cout-terme, défini comme la solution d'un problème d'optimisation, souvent désigné *Unit Commitment* (UC). Au cours des dix dernières années, les travaux présentés dans la littérature ont eu pour objectif non seulement la représentation des incertitudes sous forme de contraintes supplémentaires dans la fonction objectif, mais également la conception d'une algorithmique avancée pour atteindre des temps acceptables de résolution de cette fonction objectif modifiée y compris pour les grands systèmes. Dans ce papier nous présentons une analyse des évolutions récentes du modèle UC de façon à identifier des nouvelles tendances.

ABSTRACT – The increasing share of variable renewable energy generation (VG), as a response to environmental concerns, brings along new challenges to conciliate economics with security in power system supply. Nowadays, the more developed VG sources are wind and solar, which have low controllability and a variable output that is only partially predictable. This paper presents an overview on recent developments in Unit Commitment (UC) problems in order to take into account the uncertainty in the demand-generation balance. This subject has been widely discussed in literature over the last years in hundreds of scientific publications, mainly related to the impact of deregulation and the management of forecast errors. A wide variety of approaches to include uncertainties in conventional generation day-ahead optimization, and to represent these uncertainties has been proposed in literature. These include modifications in the objective function, enhanced security constraints and solution methods that improve computational speed. In this paper we analyse the development presented in the literature in order to identify evolution trends in UC models to achieve more robust solutions.

KEYWORDS – Stochastic unit commitment, probabilistic scheduling, scenario-based uncertainties, variable generation, mixed integer linear programming.

Nomenclature

Indices

I	Index of available units (from 1 to N)
T	Index of time intervals (from 1 to T)
S	Index of scenarios considered (from 1 to S)
M	Index of piecewise linearization (from 1 to M)
T	Index for minimal T_i^{up} and T_i^{dn} constraints

Parameters

G_i^{min}	Minimal stable generation of unit i [MW]
G_i^{max}	Maximal output power of unit i [MW]
R_i^{up}	Maximal ramp up of unit i [MW/h]
R_i^{dn}	Maximal ramp down of unit i [MW/h]
T_i^{up}	Minimal up time of unit i [h]
T_i^{dn}	Minimal down time of unit i [h]
R_{det}	deterministic reserve requirement [MW]
P_s	Probability of scenario s
a_i α_i β_i γ_i	Cost coefficients [$]
b_i c_i	Cost coefficients [$/MW] and [$/MW^2]
τ_b	Boiler time constant [h]

Cost function

C_i	generation cost of unit i [$]
S_i	start up and shutdown costs unit i [$]
S	set of forecasting errors (scenario tree)

Variables

d_f^t	forecasted net demand at time t [MW]
d^t	net demand at time t [MW]
r_i^t	reserve of unit i at time t [MW]
g_i^t	output power of unit i at time t [MW]
$x_i^{up,t}$	unit i committed at time t (1 *up*)
$x_i^{dn,t}$	unit i decommitted at time t (1 *down*)
u_i^t	state of unit i at time t (1 *on* – 0 *off*)
w_c^t	wind curtailment decision at time t (1/0)
t_i^{dn}	Off-time of unit i [h]

1. Introduction

The optimal short-term schedule of generation to supply electric power demand with a reasonable risk level has been matter of concern in utilities for at least five decades, which is why it had been widely addressed in literature [1-5]. This problem has been often designated as Unit Commitment (UC). A detailed UC model corresponds to a large-scale Mixed-Integer Non-Linear Programming (MINLP) problem, where the operating cost among all units, subject to power balance and technical constraints at every interval of the optimization period, is minimized. However, over time different approximated models have been used and intuitive methods such as Exhaustive Enumeration (EE) [6] and Priority List (PL) [7] were initially applied to small power systems. Afterward, the simultaneous growth of the electric system infrastructure and computational power drove the application of formal optimization techniques such as Dynamic Programming (DP) approaches [8] to solve the UC problem. Since the early application of DP, the mathematical formulation of the UC problem has not ceased to evolve and several bibliographic reviews have been published, some of them will be discussed to cover historical progression [1-5]. Until 2000 two main trends of evolution can be identified: modelling of new operating constraints and application of different optimization techniques.

In 1987 Cohen *et al*. studied 76 references and discussed the scope of the short-term UC problem, with emphasis in hydro-thermal constraints [1]. The available solution methods for the UC problem were: DP, Lagrange Relaxation (LR), the branch-and-bound method and Benders decomposition. In 1994 Sheble *et al*. presented a list of 80 available references for the solution of the thermal UC problem and concluded that LR was the most promising technique to solve the short-term scheduling problem [2]. Drawbacks of DP approaches were the suboptimal treatment of inter-temporal units' constraints and the need to limit the commitments considered at any hour to circumvent the combinatorial nature of the UC problem. In addition, the optimality principle may be violated when taking into account minimum up/down time constraints, which impose an on-line/off-line time once a unit has been start-up/shutdown [9].

In 2004 Padhy put together 35 years of research on UC that included over 150 published articles [3]. Three topics were discussed: UC cost minimization vs. profit maximization firsts' formulations, the inclusion of network constraints in the UC (Security-Constrained Unit Commitment SCUC) and optimization techniques. This time a new trend was identified concerning the more suitable optimization approach for UC problems: hybrid models, based on a combination of classic techniques and more innovating ones (heuristic methods). This work has been updated in [4], but comments about research trends were avoided. Finally, an exhaustive literature survey, with more than 400 references, on optimization methods applied to large-scale UC, with special attention in uncertainty consideration, has been recently published [5].

After deregulation, a new perspective appeared in the field of production resource optimization. In a market-based power system, the Generating Companies (GENCOs) objective is revenue maximization, which has driven the development of several Bidding Strategies [10], while power balance and network security responsibility is held by the System Operators (SO). Then, the cost of energy purchases is minimized (or the social welfare is maximized) through Market Clearing (MC) mechanisms, whether by the SO [11] or a Power Exchange. Nevertheless, these issues represent nowadays research topics on their own that lie outside the scope of this work.

Furthermore, power systems have been facing operational transformation due to Variable Generation (VG) integration. The additional variability and uncertainty of this type of generation has opened a new research axis related to their incorporation into the demand-generation scheduling problem. This increasing penetration of VG has raised concern about the ability of the power system to economically preserve the power balance without increasing operational risk. The use of classic security criteria in the UC formulation, in particular the deterministic N-1 reserve requirement, has been questioned in scientific publications, for both economic and security reasons [12]. Hence, different alternatives have been proposed, mainly in the last 10 years, with recent emphasis on explicit representation of uncertainties within the UC model.

In this direction, a consensus towards Mixed Integer Linear Programming (MILP) formulations seems to emerge. Arguments for this choice have been presented in literature, but a comprehensive discussion on optimization techniques applied to UC under uncertainties could be premature, for example Robust Optimization (RO) and Chance-Constrained (CCO) approaches are still to be more deeply explored. Alternatively, an overview of uncertainty considerations in UC models could help researchers and the industry to keep track of recent developments on the topic. The aim of this work is not to provide an exhaustive literature survey but to discuss the evolution of UC models with a probabilistic representation of certain aspects that impact system security. The focus of this paper is to describe MILP formulations and scenario-based representation of uncertainty. The additional computational complexity derived from the inclusion of the scenario dimension is also discussed.

Before addressing the inclusion of uncertainties in the UC model, some general concepts about the classic UC formulation, referred also as *Deterministic* UC are presented Section 2, followed by some details of its MILP formulation. Section 3 presents the basis of removing deterministic reserve constraints and ensuring a solution that co-optimize security and energy cost. Section 4 includes a synopsis about the formal inclusion of uncertainties in the optimization process from an economic perspective (cost reduction) and from a security perspective (security constraints), providing a summary table on UC under uncertainties main formulations. Finally, conclusions are drawn in Section 5.

2. Deterministic Unit Commitment (UC)

Unit Commitment is defined here as the process of optimizing production resources within a certain period (i.e. 24 hr). The outcome of this process is the state of each unit (on/off) at each time interval of the defined period. This optimization is subject to three types of constraints: power balance (2), reserve (3), and generators' technical constraints (4-8), while the objective function is the minimization of operating cost (1). For the sake of clarity, a system with perfect market competition and a thermal generation mix is presented as an example. In such a system, the components of the operating cost are energy generation, start-up and sometimes shutdown costs. The deterministic problem assumes a perfect knowledge of generation and demand profile and all uncertainty is covered by adding a deterministic reserve constraint. This constraint ensures that the solution obtained permits serving demand and holding a pre-defined volume of spare capacity that can be used to handle the uncertainties. The volume of reserve is related to the requirement of the system. It can be fixed or variable and is computed outside the optimization problem.

2.1 Typical formulation of the Deterministic UC

Objective function:

$$min_{g,u}\left\{\sum_{t=1}^{T}\sum_{i=1}^{N}[C_i(u_i^t,g_i^t)+S_i(u_i^t,u_i^{t-1})]\right\} \qquad [\$] \tag{1}$$

Power balance constraint (single bus bar with infinite network capacity):

$$D_f^t=\sum_{i=1}^{N}g_i^t \qquad \forall\ t=1,\dots T. \qquad [\mathrm{MW}] \tag{2}$$

Upward Reserve constraint:

$$\sum_{i}^{N}r_i^t\geq R_{det} \qquad \forall\ t=1,\dots T. \qquad [\mathrm{MW}] \qquad r_i^t=min\{G_i^{max}-g_i^t,u_i^tR_i^{up}\} \tag{3}$$

Units' power limits constraint: unit minimal and maximal power output.

$$u_i^tG_i^{min}\leq g_i^t\leq u_i^tG_i^{max} \qquad \forall\ i=1,\dots N.,\forall\ t=1,\dots T. \qquad [\mathrm{MW}] \tag{4}$$

Units' ramping limits constraint: maximum power that can be released from one optimization period to the following.

$$g_i^t-g_i^{t-1}\leq R_i^{up} \qquad \forall\ i=1,\dots N.,\forall\ t=2,\dots T. \qquad [\mathrm{MW}] \tag{5}$$

$$g_i^{t-1}-g_i^t\leq R_i^{dn} \qquad \forall\ i=1,\dots N.,\forall\ t=2,\dots T. \qquad [\mathrm{MW}] \tag{6}$$

Minimum up time: is the minimum time that a unit must remain on-line once it has been start-up.

$$\sum_{\tau=t}^{t+T_i^{up}-1}u_i^\tau\geq T_i^{up}x_i^{up,t} \qquad \forall\ i=1,\dots N.,\forall\ t=1,\dots T. \qquad [\mathrm{h}] \tag{7}$$

Minimum down time: is the minimum time that a unit must remain off-line once it has been shut down.

$$\sum_{\tau=t}^{t+T_i^{dn}-1}(1-u_i^\tau)\geq T_i^{dn}x_i^{dn,t} \qquad \forall\ i=1,\dots N.,\forall\ t=1,\dots T. \qquad [\mathrm{h}] \tag{8}$$

The minimum up/down time are inter-temporal constraints traditionally represented by non-linear expressions [13]. In (7)-(8) linear expressions were selected [14]. It must be noted that many others technical constraints have been included in literature. Some of them are listed in Table 1. However, their detailed consideration is beyond the scope of this work.

Table 1. References of UC additional constraints formulations

Some other constraints	Reference
Hydro-thermal systems: reservoir level, energy constraint on water used over a day, etc.	[15]
Network constraints: power flow, bus voltage, transformer tap limits, power losses, etc.	[16]
Fuel/Emission (CO2) constraints.	[17] [18]

2.2 Cost functions definition

Equation (1) includes production cost (C) and start-up and shut-down costs (S). In general, the production cost can be represented by a quadratic function (9), while start-up costs exhibit an exponential behaviour with respect to off-time (10). Shutdown costs account for fuel waste and they can be considered as constant for a specific unit (11) [19].

$$C_i(u_i^t, g_i^t) = u_i^t[a_i + b_i g_i^t + c_i(g_i^t)^2] \quad (9)$$

$$S_i^{up}(x_i^{up,t}) = x_i^{up,t}\left[\alpha_i^{boiler}\left(1 - e^{-\frac{t_i^{dn}}{\tau_b}}\right) + \beta_i^{turbine}\right] \quad (10)$$

$$S_i^{dn}(x_i^{dn,t}) = x_i^{dn,t}[\gamma_i] \quad (11)$$

An abundance of formulations for operating costs, associated to specific types of units, exist in literature. Nevertheless, a discussion on this matter is beyond the scope of this work. Expression (9), (10) and (11) were selected for illustrative purposes because they are the departure model frequently used in the MILP formulations treated in the following.

2.3 Optimization methods applied to power systems UC problem

A comprehensive literature review on optimization methods applied to solve the UC problem has been developed in [1-5]. This section focuses on the last 25 years of research, so developments based on Exhaustive Enumeration (EE), Priority List (PL) and Dynamic Programming (DP) are not considered. The attention is given to decomposition, integer and linear programming approaches. Some basic references are listed in Table 2.

Table 2. References on Optimization methods applied to the UC problem

Optimization Techniques	Relevant comment	Reference
Lagrangian Relaxation (LR) + Dynamic Programming (DP) Lagrange Multipliers (LM) calculated by the Subgradient Method	LR: Sub-problem coordination DP: single unit sub-problem solution	[20]
Benders decomposition (BD) + Integer Programming (IP)	BD: Pure-integer nonlinear UC Nonlinear Economic Dispatch (ED)	[21]
Dantzig-Wolfe decomposition (DW) + Linear Programming (LP)	LP: solution of sub-problems	[22]
Mixed Integer Linear Programming (MILP), based on branch-and-bound and cutting planes algorithms.	Cost function piecewise linearization Linearization of constraints	[23]
Evolutionary programming (EP)		[24]
Meta-heuristic Methods - Genetic Algorithm (GA) - Simulated Annealing (SA) - Particle Swarm Optimization (PSO) - Neural Network (NN) - Tabu search - Hybrid models (ex. LR+GA. GA used for updating LM)		 [25] [26] [27] [28] [29] [30]

In the 70's, LR, DW decomposition and Benders partition had been examined, followed by the used IP for the master problem and DP or LP to solve the sub-problems, due to the inability to solve large-scale centralized UC problems. DW was mostly used for single-period Economic Dispatch (ED) and Long-term UC. In the 80's, DP was used to solve the complete UC problem, but it was then replaced by LR that become the privileged optimization method for UC [2]. After 2000, published papers exhibited an interest on hybrid models, based on classic models combined with heuristic methods such as Genetic Algorithms (GA). In this example, it was stated that, although LR approaches provided fast solution with reasonable storage and computational time requirements, they might suffer from numerical convergence and solution quality problems [30], and that this drawback could be circumvent by meta-heuristic methods.

More recently (from 2005), MILP and BD approaches received a lot of attention. In [31] and [32] a comparative analysis between LR and MILP explains this turnaround well, linked to increasing computing power, development of performing MILP commercial solvers, and above all, the higher accuracy of MILP. It is worth noting that a 0.5% deviation in fuel costs can represent millions of dollars yearly in large utilities [3]. For example, the duality gap (solution quality measurement) of LR approaches used to be around 1-2 %, which was an acceptable value for industrial applications before electrical industry deregulation [4], while MILP might attain lower values.

Nevertheless, it must be acknowledged that this precision improvement comes at a price: computing time. MILP solvers may be based on the *branch-and-cut* algorithm, whose execution time grows exponentially with the size of the problem. This relation is linear for LR due to the nature of the decomposition approach. In other words, LR technique decomposes the UC problem in single-unit sub-problems, if more units are added (problem size increase) the amount of sub-problems increases linearly. In the following, only MILP formulations will be considered since it seems to be (at 2014) the most widespread optimization method for uncertainty modelling in UC formulations, although more sophisticated optimization methods, such as RO have been proposed recently [33]. The focus will be placed on the definition of the security criterion, meaning the fixed reserve, the probability or expected impact of uncertainties, etc.

2.4 MILP deterministic UC formulation

For applying MILP to solve the UC problem, the cost function and constraints must be linear with respect to the optimization variables. Classic constraints, such as (2)-(8) have already been represented by linear expressions [14], but (9) and (10) need to be linearized.

2.4.1 Cost functions linear approximation

In [23] a piecewise linearization has been proposed for (9) and a stairwise approximation is adopted for (10). In addition, this work proposed to express start-up and shutdown cost and constraints as a function of state variables (u), to avoid auxiliary variables for start-up/shutdown decisions (x) and reduce computation time. Moreover, the different constraints enumerated in Table 1, and some others, have been subsequently included in MILP formulations, for example: network consideration through DC power flow [34], hydroelectric plants reservoir constraints [35], etc.

2.4.2 Computational complexity

The optimization model represented by equations (1)-(8) has been implemented in MATLAB and solved using a commercial Mixed Integer Linear Programming (MILP) package: Gurobi [36]. For illustrative purposes, a test system of 10 units, whose parameters can be found in [25], has been optimized for a 24 hours period with an hourly step, considering only constant and linear incremental fuel costs (to avoid piecewise and stairwise approximation).

The optimization variables are the state of each unit (binary) and its output power (continuous), at each interval. Start-up and shut-down auxiliary binary variables are also defined for each unit and interval. Thus, this simple system involves an optimization problem of 240 continuous variables and 720 binary variables. Regarding the constraints, power balance and reserve are imposed at each hour (48), while maximal and minimal power (480), ramping limits (460) and minimum up/down time (480), relation between start-up, shutdown and state variables (470) are specifics of each unit, so a total of 1938 linear constraints is achieved. Obviously, the constraint matrix is sparse, and the solver pre-solving algorithm is able to reduce the size of the problem removing inactive constraints. With current computing power and performing commercial solvers this problem was solved on a personal computer (Intel Core i5, 2.8 GHz with 16 GB of RAM) in only a few seconds (approx. 2 s).

In general, increasing computational power and developments on optimization techniques have enabled to overcome historical challenges in UC problems; nevertheless, new ones appear, driven by the need of a more suitable representation of ever-changing power system operational environment. In particular, the optimization of spinning reserves, and its consideration within UC models, has received a lot of attention as it is discussed below.

3. Spinning Reserve optimization

Spinning Reserve (SR) is the difference between the sum of the power ratings of all the operating units and their actual load, if the reader is willing to overlook ramping constraint details for the purposes of this section. It will assure power balance following VG and load forecasting error, and unit failures. Depending on response time and its main function, SR can be classified into primary, secondary and tertiary reserve. In general, Primary Reserve (PR) is meant to avoid load shedding after sudden disturbances and stabilize power system frequency in seconds (approx. 30 s) [37], but it does not permit to bring the frequency back to the reference value. A slower reserve, the secondary reserve, is then used for bringing the frequency back to its nominal value in some minutes. It also allows compensating for very short term demand variability. Finally, Tertiary Reserve (TR) is mainly conceived as a replacement reserve, to rebuild deployed PR and SR. In addition, TSOs also dispose of Quick Start (QS) units that can be brought on-line in several minutes as a Non-Spinning Reserve (NSR) also called standing reserve. In this section, only spinning TR is discussed.

3.1 Why has the fixed SR requirement been questioned?

SRs have traditionally been defined outside the UC optimization using simple risk criterion such as the N-1, meaning the capacity of the largest unit. However, increasing forecasting errors, linked with VG, might augment the deployment of reserve during load following, leaving the system exposed in case of unit faults, or even introduce ramping events. These concerns have been treated from two perspectives: SR requirement (volume) and flexibility. This work deals with the former, considering the fact that reserving on-line capacity leads to additional cost (ex. schedule sub-optimization).

3.2 Risk based approaches

An alternative way to redefine SR requirement is to fix a risk threshold over the schedule horizon, for example a reliability index such as the Loss of Load Probability (LOLP) [38]. In this case units' failures probabilities can be represented through a Capability Outage Probability Table (COPT) and required SR for constant risk is determined for each time interval. Several approaches to include risk constraints in UC model appeared, for example in [12] a methodology to determine iteratively the minimal SR requirement in a daily schedule, within the UC calculation, was proposed. A suitable technique for solving this kind of approach is the Chance Constrained Optimization (CCO); however it has not been widely deployed in literature for short-term scheduling, but mostly for planning purposes [5].

The UC risk constraint may be defined at each time step individually or through the whole daily production plan. In the first case the problem is formulated as an Individual CCO program (ICCO). On the other hand, if the risk is defined through the entire optimization horizon, a Joint CCO (JCCO) problem is to be solved. In practice, only the JCCO approach assures the desired safety level, but it is more difficult to solve [39], which has led to ICCO approximation.

Drawbacks of risk-based methods have been widely discussed in literature; for example, it has been argued that the definition of risk targets for a specific system under any operation condition might result arbitrary and suboptimal. In addition, this approaches do not consider that at each demand level the cost of providing reserve may be different, i.e. at low load, reserve is provided by partially load cheap units, while at peak load, expensive units must be committed to supply reserve. Therefore, they have been partially displaced in research literature by cost/benefit approaches. However, it is worth noting that the fixed risk criterion remains the current practice in real system applications.

3.3 Cost/Benefit approaches

In general, cost/benefit approaches are based on a bi-objective optimization, which means that the fixed risk is replaced by an *expected benefit value* that is added to the objective function as a cost, for example, the Expected Energy Not Supplied (EENS) multiplied by the Value of Loss Load (VOLL). One simple way to do this is through a bi-level optimization model, where a program, based on classical UC cost functions and constraints, determines the optimal spinning reserve requirement at each time interval, neglecting inter-temporal constraints [40]. Then, the solution can be fed into a classic UC formulation, in which the reserve requirement is specific to each hour (load level). In [40] only units' availability was considered but then forecasting errors of load and wind generation were included [41].

This approach can be considered as a *sequential* optimization of reserve and energy, in contrast to simultaneous ones, where the SR is optimized within the UC model. The latter have been mostly developed in bidding strategic and MC mechanisms where reserve is remunerated in the market [42].

4. Including uncertainty in UC models

The inputs of the UC optimization process are inherently uncertain, which make that the decision taken might no longer be the optimal in real time; for example, if VG was overestimated and not enough capacity was committed, expensive units QS may be needed. Therefore, uncertainty consideration in UC model may produce more robust solutions.

4.1 Where the uncertainties come from?

In reality, uncertainties in UC models may come from different sources, including units' availability, demand and VG forecasting errors, or even units' parameters (cost coefficients, ramping limits, etc.). Traditionally, only unit availability and demand forecast errors were considered. Nowadays, VG forecasting errors are added to demand ones, so the uncertainty in net demand (load less distributed generation) becomes more and more important with the increasing share of this kind of source. After deregulation, a new level of uncertainty, which comes from the market organization, is added to the decision process of GENCOs that maximize their profit based on energy price prevision [43]. In the following sections, only publications considering unit unavailability and net demand forecast errors will be analysed. Issues related to profit maximization under price uncertainty are not included, since this subject is mainly related to bidding strategies and MC mechanism developments, which depend on market structure.

4.2 How to model them?

Then, these uncertainties have been represented by at least 3 types of variables; some examples are given in Table 3.

Table 3. Uncertainty representation

Type of variable	Interval number	Fuzzy	Random
Technique	Interval Optimization (IO)	DP, MILP, SA, PSO…	Stochastic Optimization (SO), BD

It can be observed that some classical optimization techniques used in deterministic UC, such as DP [44], MILP [45] and meta-heuristic methods reappears, mainly in Fuzzy logic approaches. Also new techniques are proposed to deal with uncertainties. In particular, IO, as a specific case of Robust Optimization (RO), impose limits to the uncertain variables, without specifying any distribution law; then the operating cost can be bounded, taking into account extreme case [46]. In the following only probabilistic representation (random variables), more specifically scenario-based MILP formulations, will be discussed, since the attention is placed on the treatment of the security requirement. The scenarios might be generated analytically from discretisation of probability laws [47] to generate the scenario tree in stochastic programming, or through sampling methods such as Monte Carlo simulation from distribution laws or time series [48].

4.3 Uncertainties in scenario-based UC optimization models

At least three approaches for co-optimization of energy and reserve levels under uncertainties on a daily basis can be identified in literature. These general methodological concepts may be applied to MC or UC models.

From cost/benefit approaches, one option considers the minimization of the *expected* production and curtailment cost over a scenario set through Stochastic Optimization (SO). In [33] a stochastic MC was formulated. However, the inclusion of a probabilistic index in the objective function results in a high computational burden when considering continuous uncertainties, such as forecasting errors [47]. One alternative consists on including a probabilistic security constraint (CCO), instead of a penalization in the cost function [49]. Finally, another option considers a two stage optimization through a decomposition approach [50]. A considerable amount of research work has been carried out on this subject and several publications for these approaches, including different optimization techniques, were found in literature [5].

4.3.1 A Cost/Benefit Stochastic Optimization (SO)

In the deterministic UC the optimization vector contains one state variable for each unit and time interval. The consideration of scenarios adds a new dimension to the optimization problem, which means that one state variable, is defined for each unit, each hour *and each scenario*. Then, the sum of production costs over all scenarios, weighted by their probability (P_s), is minimized. This can be expressed in a simplified way as (12).

$$min_{g,u}\left\{\sum_{s\in S} P_s \sum_{t=1}^{T}\sum_{i=1}^{N}\left[C_i\left(u_i^{t,s}, g_i^{t,s}\right) + S_i\left(u_i^{t,s}, u_i^{t-1,s}\right)\right]\right\} \tag{12}$$

Involuntary load shedding and VG curtailment are often included as optimization variables to assure feasibility and their expected costs is penalized in the objective function (cost/benefit approach) [33] [47]. It is worth noting that only one decision must be the output of the optimization problem, so a set of constraints will impose the equality of state variables over scenarios; they are usually referred in literature as *non-anticipative constraints*. In this case, the NSRs are not taken into account. To get a picture of the computational complexity of this problem, let's consider demand forecasting errors in the system presented in section 2.5, represented by a Gaussian distribution discretized in 7 steps. The scenario tree over 24 hr will contain over $1.9\text{x}10^{20}$ scenarios (7^{24}). Obviously, inter-hour correlation of wind, for example, limits the possible transitions, hence scenario reduction techniques had also received a lot of attention [33].

It has been stated that this kind of approach achieved lower costs than deterministic ones. However, this is due to the inclusion of the EENS cost, which depends on the specified VOLL. If a low value is attached to curtailment (VOLL) this approach may provide a cheaper schedule but with a higher risk. On the other hand, if a high VOLL is considered, more reserve might be committed and the stochastic schedule will be more expensive. Therefore, deterministic and stochastic UC cost cannot be directly compared. One alternative proposed in literature is to consider a *perfect forecast* scenario to define a lower bound to production cost and provide a reference to compare different UC policies [48] [51].

4.3.2 A new probabilistic security constraint

Hybrid deterministic/probabilistic approaches, where traditional N-1 constraint (2) is enforced together with a probabilistic one (CCO), have been proposed, since stochastic UC results computational intractable for realistic system. One option is to limit the probability that the scheduled reserve is exceeded by net demand forecasting errors [49]:

$$Prob\left\{-\sum_{i}^{N} r_i^{dn,t} \leq d^t(w_c^t) - d_f^t \leq \sum_{i}^{N} r_i^{up,t}\right\} \geq 1-\varepsilon \qquad \forall\, t = 1,2,\dots T. \tag{15}$$

This way, the scenario dimension is avoided in the objective function and the new security constraint will be active only when the deterministic reserve constraint (meant to face unit outages) fails to handle forecasting errors. In that case, more reserve is scheduled at a higher production cost. It must be noted that (15) was defined independently for each time interval (ICCO) and only wind generation forecasting errors were considered as an uncertainty source. Wind curtailment was explicitly modelled as a decision variable (w_c^t) that reshaped net demand distribution [49]. Stochastic and hybrid approaches will be referred as *integrated* models, in contrast with decomposition ones, since they compute simultaneously the units' state and reserve levels accounting for uncertainties within a global UC MILP formulation.

4.3.3 Two stage model (Bender Decomposition)

The idea is to optimize the *forecasted* cost in a first stage and then penalize *expected* constraint violations found in a second stage. In [50] a classic UC is determined from demand and intermittent generation forecasts in a first stage. Then, scenarios are generated from Monte Carlo simulation and the limits of corrective actions are verified through simulation of real-time redispatch. If violations are found, bender cuts are generated and added to the initial UC formulation. It is concluded that a more expressive schedule is achieved; this is the price of security. However, the used of bender decomposition approach is based on the fact that a cutting planes can be actually computed [52].

4.4 Summary of uncertainty inclusion in UC model

Table 4 summarized main considerations discussed in this section, identifying the source of uncertainty modelled, the scenario construction technique and the optimization approach used to include the security penalty in some representative references.

Table 4. Matrix of consideration sets of aforementioned proposals for UC under uncertainties

Reference	MILP Optimization approach			Represented uncertainty			Scenario generation		Security Penalty
	Sequential	Integrated	Decomposition	Load	Wind	Availability	Analytic	Sampling	
[40]	X					X	COPT		External
[41]	X			X	X		X		External
[33]		X				X	COPT + Line outage		Cost function
[47]		X		X	X		X		Cost function
[49]		X		X	X		X		Constraint
[50]			X		X			MCS	Bender Cuts

5. Conclusion

The UC problem has been widely discussed in literature, and research has evolved in different directions over time. The increasing amount of scientific publication in the last 10 years proves that it is currently an active research field, even after five decades of developments. Moreover, further work still remains in order to respond to the needs of real power systems, which are facing a transformation in their operational environment. The consideration of uncertainties in UC models is dimensionally cursed and important effort is concentrated in the reduction of the computational burden, for example through important sampling techniques. Hence, the developments required to produce and solve robust UC models under uncertainty call for multi-disciplinary competences, such as power system economics and security, physical and operational constraints, probabilistic and stochastic modelling, optimization, etc. This work has attempted to give a compact introductory framework for uncertainty management in UC models, putting together:

- A discussion on power system operational environment evolution: deregulation and VG integration, which has driven the new UC formulations, mainly due to security concerns (reserve vs. uncertainty);
- A basic mathematical formulation of the simplest thermal UC, including reference for additional constraints;
- New trends in optimization methods applied to the UC problem, including discussions on historical drivers;
- An overview of published UC models including uncertainties, with discussion on their sources and representation.

Regarding security consideration, this work has reviewed some approaches dealing with how to dimension (or optimize) reserve to produce a secure schedule when VG forecasting errors are added to demand ones and to units' unavailability. It has been shown that it depends on the probability of overlapping events and how much value is attached to *security*. However, until now, the acceptability criterion to define a *secure schedule* has been based on on-line capacity sufficiency over scenarios [33] [40]. In [49] a security constraint was defined, but once more it only limited the probability that forecasting errors exceed the reserve volume available at each time step. In [50] redispatch and network limits were verified based on a ten minute ramping constraint (derived from the hourly ramping).

Further work will focus on the development of a new constraint able to represent not only the volume of reserve over scenarios, but also the *quality* of this reserve from a dynamic security point of view. The reduction of inertia due to the replacement of synchronous conventional generators by VG connected through power electronics deteriorates the dynamic response of electric systems following units' faults. This could lead to under-frequency load curtailment in small power systems with an important share of VG. In this case, load shedding might be prevented by consideration of units' primary regulation capabilities, for instance, while optimizing the generation schedule.

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